Sparse Complete Orthogonal Factorization as Applied to Bistatic Target Strength Prediction

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Abstract. In this work we present a complete orthogonal factorization method that takes advantage of the structure and numerical properties in sparse, rank deficient, overdetermined systems arising in bistatic target strength prediction models. The sparse overdetermined matrices in these models belong to a class called row-bordered block diagonal matrices. Applying this decomposition method to highly ill-conditioned bistatic target strength prediction models, we have obtained at least ten (10) fold improvement over the compute time required by the linear algebra package LAPACK for a range of models. Further speedups can be realized by exploiting parallelism inherent in the models. Algorithmic speedups of this magnitude will enable the extension of the frequency range in bistatic target strength prediction models to regions of greater interest to the Navy. All computer runs were done on the NCCOSC RDT&E Division Convex Exemplar.

1. Introduction. Knowledge of bistatic target strength is of increasing importance in sonar systems. Full-scale measurements of monostatic target strength are expensive and difficult, and become impractical for general bistatic geometries. Usually one is limited to measurements at a limited number of monostatic angles and perhaps a few bistatic angles in a single plane. Extending the value of this measured data by using it to estimate the full bistatic target strength pattern is extremely desirable. Previous efforts to do this have severe limitations or restrictions in their applicability.

A method has been developed [1] that relies on measured monostatic and limited bistatic data, and uses a numerical model to estimate the surface field and propagate it to the farfield for full bistatic geometries. The crux of this method is to construct the farfield propagator matrix that relates the farfield scattered pressures to some surface quantity. A singular value decomposition of the propagator matrix is used to eliminate nonradiating surface modes. Then the radiating part of the surface values is determined by least squares approximation from knowledge of the measured monostatic and limited bistatic scattering data and from the principle of reciprocity, that requires the scattering matrix to be symmetric. The least squares problem involves a large, sparse, rank deficient matrix. Although the solution of this least squares problem is nonunique, it appears that the minimum norm solution gives good results when the surface values are used to reconstruct the full bistatic scattering pattern. Available conventional algorithms, such as those contained in LAPACK, deal with the rank deficiency, but take no advantage of the sparsity. Therefore, they are very time consuming, and this severely limits the frequency range of applicability.

2. Sparsity structure of matrix in the bistatic target strength prediction model. Large sparse overdetermined systems of equations $Mx = b$ arise in numerous scientific and engineering applications. In the bistatic target strength prediction (btsp) model, the sparse overdetermined matrix M is rank deficient , and also, for some permutation matrix P, PM is a 2-by-1 block matrix

$$PM = \begin{pmatrix} A \\ G \end{pmatrix},$$

where A = [

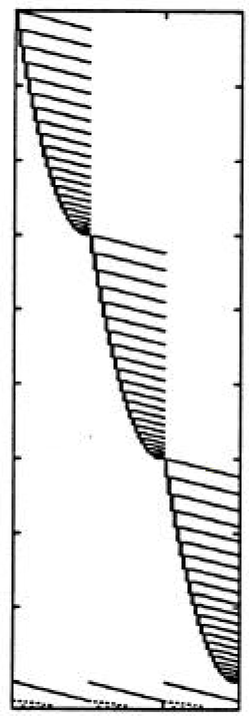

Fig. 1. Row-bordered block diagonal form PM of a 935-by-300 matrix M.

where T_{11} is an r-by-r upper triangular matrix and nonsingular. Making use of this relation in $Q^T MS$ gives the so-called complete orthogonal factorization

$$MS = Q\begin{pmatrix} T_{11} & 0 \\ 0 & 0 \end{pmatrix} Z^T .$$

In terms of the orthogonal matrices Q and Z and the permutation matrix S, the overdetermined system Mx = b has the equivalent form

$$(Q^T MSZ)(Z^T S^T x) = (Q^T b).$$

Combining these last two equations gives

$$x = SZ\begin{pmatrix} T_{11}^{-1} c \\ 0 \end{pmatrix},$$

which is the minimum norm solution to the rank deficient linear least squares problem as computed in LAPACK [3].

4. A linear least squares solution that takes advantage of block diagonal form of A**.** When QR factorization with column pivoting is applied to the row-bordered block diagonal matrix PM, the leading block A in PM loses its block diagonal form after only a few Householder updates. This deformation in the block diagonal form of A is a direct consequence of the fact that each row and each column in every block of the 1-by-k block matrix G in PM contains at least one nonzero entry.

In this section, we present a method that takes advantage of the block diagonal form of A to compute a linear least squares solution of the overdetermined system Mx = b. The first and initial step uses QR factorization with column pivoting to factorize each of the k rank deficient diagonal blocks of A individually. At the completion of this step, we obtain an orthogonal (or unitary) matrix $Q^{(i)}$ and an n-by-n permutation matrix $S^{(i)}$ such that

$$A_{ii} S^{(i)} = Q^{(i)} \begin{pmatrix} U_{11}^{(i)} & U_{12}^{(i)} \\ 0 & 0 \end{pmatrix},$$

where $U_{11}^{(i)}$ is an s_i-by-s_i nonsingular upper triangular matrix with $s_i = \mathrm{rank}(A_{ii})$, for i = 1,..., k. If block A_{ii} is of full rank, then we have

$$A_{ii} S^{(i)} = Q^{(i)} \begin{pmatrix} U_{11}^{(i)} \\ 0 \end{pmatrix}.$$

Since the original matrix M in the btsp model is rank deficient, at least one diagonal block of A must be rank deficient. In practice, we have observed that each and every diagonal block of A is rank deficient, and so we will assume henceforth that all diagonal blocks of A are rank deficient.

Let us define

$$G_{1i} S^{(i)} = \begin{pmatrix} U_{21}^{(i)} & U_{22}^{(i)} \end{pmatrix} \qquad i = 1,..., k,$$

and use the orthogonal matrices $Q^{(i)}$ and permutation matrices $S^{(i)}$ to form the following matrices

$$Q_1 = \begin{pmatrix} Q^{(1)} & & & \\ & \ddots & & \\ & & Q^{(k)} & \\ & & & I_\delta \end{pmatrix} E,$$

and

$$\Pi = \begin{pmatrix} S^{(1)} & & \\ & \ddots & \\ & & S^{(k)} \end{pmatrix} F,$$

where I_δ is a δ-by-δ identity matrix and E and F are m-by-m and n-by-n permutation matrices respectively. Then it can readily be shown that for some E and F the matrix $PM\Pi$ takes the following block form

$$PM\Pi = Q_1 \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \\ 0 & 0 \end{pmatrix},$$

where

$$U_{1j} = \begin{pmatrix} U_{1j}^{(1)} & & \\ & \ddots & \\ & & U_{1j}^{(k)} \end{pmatrix},$$

and

$$U_{2j} = \begin{pmatrix} U_{2j}^{(1)} & \ldots & U_{2j}^{(k)} \end{pmatrix} \qquad j = 1, 2.$$

The construction of the Householder update $Q_1^T(PM\Pi)$

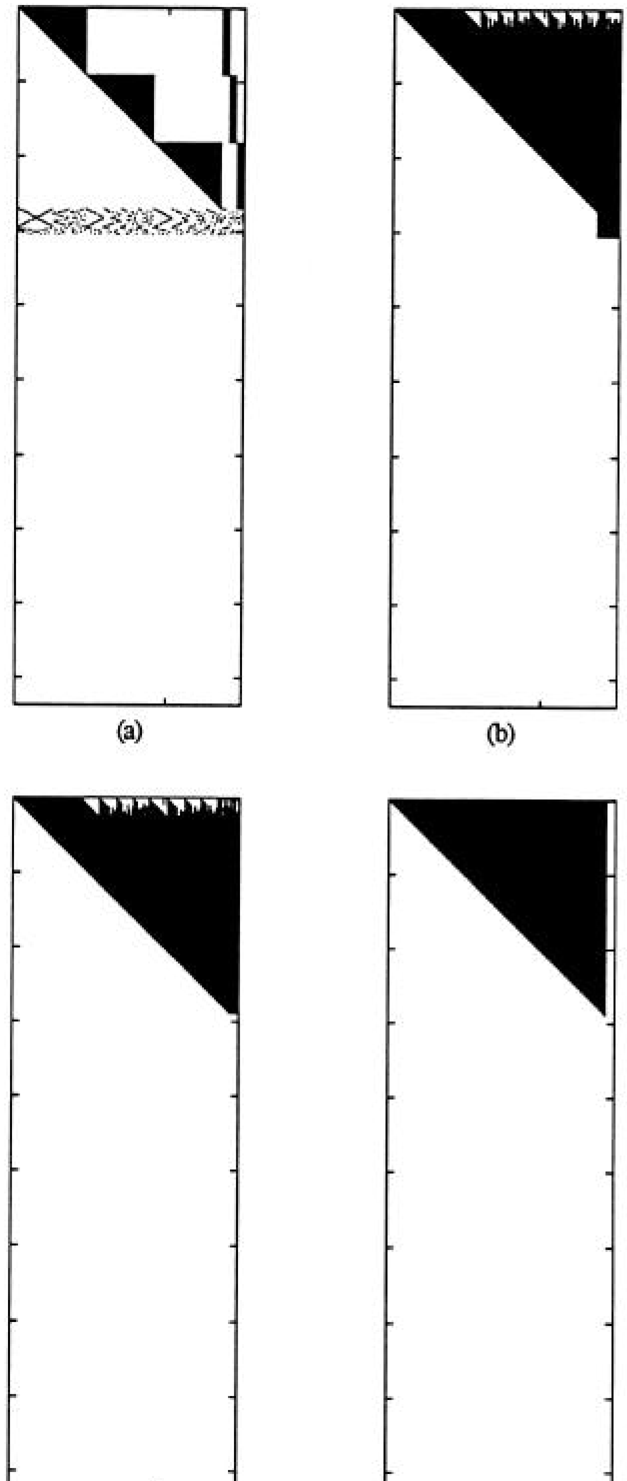

Fig. 2. (a) Householder update $Q_1^T(PM\Pi)$. (b) Householder update $(Q_1Q_2)^T(PM\Pi)$. (c) Householder update $(Q_1Q_2Q_3)^T(PM\Pi)$. (d) Householder update $(Q_1Q_2Q_3)^T(PM\Pi)Z$.

$$PM\Pi = Q_1 Q_2 Q_3 \begin{pmatrix} R_{11} & R_{12} \\ 0 & 0 \end{pmatrix}.$$

where Q

of these k Householder updates is rich in level-2 operations of matrix-vector multiplications followed by an outer product update.

6.2. Computation of Householder update Ψ_2

References

[1] H. A. Schenck, G. W. Benthien, and D. Barach, " A Hybrid Method for Predicting the Complete Scattering Function from Limited Data," J. Accoust. Soc. Am., 98, pp. 3469-3481, 1995.
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